Stoever Glass & Co
30 Wall Street, New York, NY10005

Bond Arithmetic And The Theory Behind It

This educational article explains a very complicated subject in easy to understand plain English.

To make this difficult subject easier to understand we have used 10% yields and $1,000 par value examples but no matter what the yield figures are, the principals they represent remain exactly the same.

Just as when you first were expected to find the area of a circle from its radius, so the math behind bonds can seem mystifying, even intimidating. It is true that it's complex, but let us take you through it step by step and you'll see it's not as thorny as it initially looks.

When a municipal bond is issued, there are always three fixed elements: (1) Coupon rate, (2) maturity date, and (3) value at maturity (always $1,000 per bond for municipals). All three are permanently established for the bond when it is issued and they never change. But the bond's yield to maturity can and does change as the market changes, and that causes the bond's price to change. (As you read this, it is very important to remember that a bond's price is determined by the yield to maturity and not vice versa.)

Here's how it works; taking the coupon and maturity into account, the trader assigns a yield to the bond that makes it competitive with other offerings available in the market. When that is done, we know (1) coupon rate, (2) maturity date, (3) value at maturity, and now (4) yield to maturity. At this point we apply the industry-wide formula used to translate these four elements into a price.

To understand how the formula works, remember it is evaluating an investment in which a yield is received over a period of time. And the old expression "time is money" could never be truer than when talking about yields and prices (money earning money). The concept that mathematically links yields and prices is known as the present value theory. Ask yourself: who would choose to receive $1,000 next year rather than $1,000 tomorrow? Obviously, $1,000 received tomorrow can be "put to work" for the remaining 364 days of the year and thus be worth $1,000 plus 364 days interest at the end of the year.

Now reverse your thinking. Given the value at some future date (again, always $1,000 per bond in the case of municipals), pick a yield that will be competitive with other rates and then try to determine the discounted price, or premium price you must pay today that will give you a pre-selected yield by a specified future date. How much for example, would you have to invest today at 10% compound interest to get $1,000 in five years? That would be relatively easy to answer if a bond had no coupons.

But most municipals have coupons, and the coupon rate must be taken into account. Because as we vary the coupon, the prices will vary. So to show you what the numbers look like, we've reproduced a few pages from the Bond Calculation Tables using four different coupon rates.

Four Different Bonds: Four Equal Returns

Now using these tables, let's answer this important question, which may appear deceptively self-evident: "Wouldn't I be better off getting a high coupon and reinvesting it rather than settling for a low coupon with a capital gain at maturity?"

If it's compounding your interest you're wondering about, the answer is no. A par bond, a discount bond, a premium bond and even a zero coupon bond will have the exact same rate of return when the yield to maturity is equal, assuming interest rates remain level so we can continue to re-invest coupon income at the original yield over the life of the bond.

First, let's look at Table I, the 10% coupon bond. This 10% bond, purchased at par value ($1,000), is due in five years. The income produced from the semi-annual coupons is $500.00 ($50.00 every six months). But if you could immediately reinvest each coupon at 10%, your total income over five years would be $628.92. The total five year return would be 62.89% ($628.92/$1,000).

Table I			**10% Coupon**	
Years & Months To Maturity	**Yield to Maturity**	**Price**	**Coupon Income Compounded @ 10%**	**Annualized Compounded Appreciation @10%**
5-0	10%	$1,000.00	**YOU BUY BOND NOW**	
4-6	10%	$1,000.00	$50.00	$.00
4-0	10%	$1,000.00	$52.50.	$.00
3-6	10%	$1,000.00	$55.13	$.00
3-0	10%	$1,000.00	$57.88	$.00
2-6	10%	$1,000.00	$60.78	$.00
2-0	10%	$1,000.00	$63.81	$.00
1-6	10%	$1,000.00	$67.01	$.00
1-0	10%	$1,000.00	$70.36	$.00
0-6	10%	$1,000.00	$73.87	$.00
Maturity		$1,000.00	$77.58	$.00
Total Income			**$628.92**	**$.00**
Total Return	**$628.92 (Compound income)**			**= 62.89%**
	$1,000.00 (Original investment)			

In Table II, we have a 5% bond due in five years with a 10% yield to maturity. This bond initially costs $806.95. With the 5% coupon bond, you'd receive $250.00 in coupon income ($25.00 every six months). Once again, if you were able to reinvest each coupon payment immediately at 10%, your total income over the five years would be $314.42. Add in your compounded appreciation over the life of the bond, of $193.05, and you'd have a total of $507.47. When you divide this by the price of the bond, your total return again comes to 62.89% ($507.47/$806.95).

Table II Years & Months To Maturity	Yield to Maturity	Price	**5% Coupon**	
			Coupon Income Compounded @ 10%	Annualized Compounded Appreciation @ 10%
5-0	10%	$806.95	**YOU BUY BOND NOW**	
4-6	10%	$822.30	$25.00	$15.35
4-0	10%	$838.41	$26.25	$16.11
3-6	10%	$855.34	$27.57	$16.93
3-0	10%	$873.10	$28.94	$17.76
2-6	10%	$891.76	$30.39	$18.66
2-0	10%	$911.35	$31.90	$19.59
1-6	10%	$931.91	$33.51	$20.56
1-0	10%	$953.51	$35.18	$21.56
0-6	10%	$976.19	$36.93	$22.68
Maturity		$1,000.00	$38.75	$23.81
Total Income			$314.42	$193.05
Total Return	**$314.42 + $193.05 (Compound Income) (Compound Appreciation)**			**= 62.89%**
	$806.95 (Original Investment)			

*Table II may or may not have tax consequences.

In Table III, we've gone one step further by using a zero coupon bond due in five years. In this case there is no coupon income. All you have is the appreciation of principal. That appreciation comes to $386.09 over the five years based on a purchase price of $613.91. And what's your real net return on the zero coupon? You guessed it 62.89% ($386.09/$613.91).

Table III Years & Months to Maturity	Yield to Maturity	Price	**0% Coupon** Coupon Income Compounded @10%	Annualized Compounded Appreciation @10%
5-0	10%	$613.91	**YOU BUY BOND NOW**	
4-6	10%	$644.60	$.00	$30.69
4-0	10%	$676.83	$.00	$32.23
3-6	10%	$710.68	$.00	$33.85
3-0	10%	$746.21	$.00	$35.53
2-6	10%	$783.52	$.00	$37.31
2-0	10%	$822.70	$.00	$39.18
1-6	10%	$863.83	$.00	$41.13
1-0	10%	$907.02	$.00	$43.19
0-6	10%	$952.38	$.00	$45.36
Maturity		$1,000.00	$.00	$47.62
Total Income			$.00	$386.09
Total Return	**$386.09 (compound income)**		**= 62.89%**	
	$613.91 (original investment)			

With the 12% premium bond in Table IV, you'll earn the most income: $754.69 during the five years to maturity. But you also paid the most for this bond -$1,077.21- since it was selling at a premium. In this case, the principal amortizes to par, so you have to subtract the $77.21 premium from the total income ($754.94 - 77.21 = $677.48) But, even after doing so, your rate of return is the same 62.89% that you'd get with the par, discount, and zero coupon bonds ($677.48/$1,077.21).

Table IV Years & Months to Maturity	Yield to Maturity	Price	**12% Coupon** Coupon Income Compounded @ 10%	Annualized Compounded Appreciation @ 10%
5-0	10%	$1,077.21	**YOU BUY BOND NOW**	
4-6	10%	$1,071.07	$60.00	$(6.14)
4-0	10%	$1,064.63	$63.00	$(6.44)
3-6	10%	$1,057.86	$66.15	$(6.77)
3-0	10%	$1,050.75	$69.46	$(7.11)
2-6	10%	$1,043.29	$72.93	$(7.46)
2-0	10%	$1,035.45	$76.58	$(7.84)
1-6	10%	$1,027.23	$80.41	$(8.22)
1-0	10%	$1,018.59	$84.43	$(8.64)
0-6	10%	$1,009.52	$88.65	$(9.07)
Maturity		$1,000.00	$93.08	$(9.52)
Total Income			$754.69	$(77.21)
Total Return	**$754.69 - $77.21 (compounded income) (accelerated amortization)**			**= 62.89%**
	$1,077.21 (original investment)			

The fact that we wound up with the same return in all four examples after compounding the coupon income conclusively proves that, with discount bonds, your appreciation of principal automatically compounds for you at the rate of the original yield to maturity. If it didn't, the 0% bond, with nothing but principal to compound, couldn't possibly give you a return equal to the 10% bond. Nor could it equal the 5% bond, or the 5% bond the 10% bond, or the 10% bond the 12% for that matter.

To see it happen, look at the last columns in Tables II and III. Notice how the bonds' values increase at an accelerated pace as they approach maturity. That's compounding you're seeing: (money earning money).

Tables II, III and IV also show you the present value theory in action. As you see, to get 10% a year for five years with the help of a 5% coupon, you pay $806.95 today (table II). To get the same 10% in the same time, with no help from a coupon, you need the lower price of $613.91 (Table III). In order to wind up with a 10% yield from a 12% bond, you must pay over par $1,077.21 (Table IV).

With the premium bond you have the exact same theory at work. But since the coupon is greater than the yield, the principal must be amortized to get the 10%. The formula automatically compounds or amortizes the principal at the rate of the yield to maturity, no matter what the coupon is.

A Word of Caution

But amortization of principal is no reason to shy away from premium bonds. Because what's the difference if you invest $613.91, $806.95, $1,000.00 or $1,077.21 at 10%? Your return is the same.

In each example, these are the amounts you invest today, with interest and /or principal compounded at 10% to get back $1,000.00 five years from now. And that's The present value theory in a nutshell.

Don't draw any conclusions from these examples as to which type of bond is the best buy for you.

Remember, we deliberately set all the yields at 10% to help us more clearly demonstrate how bond prices are calculated and how the formula compounds or amortizes the interest within your principal. In the real world, where marketability counts, the bonds used in these examples would not yield the same. Premiums, usually offer the highest net yields, followed by discounts, with the par bonds usually yielding the least.

Because to make one bond compete in the market with another, many factors must be taken into account. For example, a premium bond will always yield more than an identical par bond. That's because the word premium or the fact that you pay more than the bond's par value creates the wrong impression and therefore, a reluctance on the part of many investors. To help overcome that resistance, premiums offer more yield than the no-fuss-no-muss- par bonds. And that creates an opportunity for the investor who takes the time to fully understand premiums.

But now we're getting into another subject. The type of bond you buy should depend upon your individual circumstances. Just as banks and other institutions have reasons for buying what they buy, be sure you know the reasons you buy what you buy.

If you have any questions concerning the type of bonds that may be most suitable for you, feel free to discuss the matter with any Stoever Glass representative.

All of our representatives are trained to give you comprehensive explanations of how to construct a portfolio of bonds with the combination of maturities, coupons, and quality that best suit your investment objectives.

Although the above information and statistics are not guaranteed, they have been obtained from reliable sources and are believed to be accurate. See Risk Factors.